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Math 318 - Mathematical Logic

17/17

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Assignment #2

#1 How many functions are there :

Recall: $f: X \to Y$
To be a fnct, no elt in X can have more than one value in Y.
Hence, [# of fnct $f: X \to Y$] $= |Y|^{|X|}$

(1) from 2 to 3

$f: \{0,1\} \to \{0,1,2\}$
$|Y| = 3$
$|X| = 2$
numb of fncts $= 3^2 = 9$

(2) from 4 to 2

$f: \{0,1,2,3\} \to \{0,1\}$
$|Y| = 2 \quad |X| = 4$
number of fncts $= 2^4 = 16$

(3) from 5 to 1

$f: \{0,1,2,3,4\} \to \{0\}$
number of f $= 1$ (obviously, all 5 elts will be mapped to 0)

(4) from 5 to 0

$f: \{0,1,2,3,4\} \to \emptyset$
No such a function

(5) from 0 to 5

$f: \emptyset \to \{0,1,2,3,4\}$ "null function"
If we take $5^0 = 1$, there should be 1 fnct but this is a bit counter intuitive, it might also be no functions at all

#2 Are the following sets equinumerous?

(1) $[0,1)$ and $\mathbb{Q}$

From class notes

$[0,1) \sim 2^{\mathbb{N}}$ and $\mathbb{Q} \sim \mathbb{N}$

Since $2^{\mathbb{N}} \nsim \mathbb{N}$, then $[0,1)$ and $\mathbb{Q}$ are not equinumerous

(2) $[0,1]^{\mathbb{N}}$ and $[0,\infty)$

$[0,1]^{\mathbb{N}} \sim (2^{\mathbb{N}})^{\mathbb{N}} \sim 2^{\mathbb{N}\times\mathbb{N}} \sim 2^{\mathbb{N}}$

$[0,\infty) \sim \mathbb{R} \sim 2^{\mathbb{N}}$

$\Rightarrow [0,1]^{\mathbb{N}} \sim [0,\infty]$: equinumerous

(3) $[0,1]^{\mathbb{N}}$ and $\mathbb{Q}^{\mathbb{N}}$

From (2): $[0,1]^{\mathbb{N}} \sim 2^{\mathbb{N}}$

for $\mathbb{Q}^{\mathbb{N}}$: This obviously is not a countable set since:

$\#2^{\mathbb{N}} \leq \#\mathbb{N}^{\mathbb{N}} = \#\mathbb{Q}^{\mathbb{N}}$ and $2^{\mathbb{N}}$ is not countable.

So, $2^{\mathbb{N}} \leq \mathbb{N}^{\mathbb{N}} \leq (2^{\mathbb{N}})^{\mathbb{N}}$ since $\mathbb{N} \leq 2^{\mathbb{N}}$

but $(2^{\mathbb{N}})^{\mathbb{N}} = 2^{\mathbb{N}\times\mathbb{N}} = 2^{\mathbb{N}}$

Therefore, $\#2^{\mathbb{N}} \leq \#\mathbb{N}^{\mathbb{N}} \leq \# 2^{\mathbb{N}}$

$\Rightarrow \#\mathbb{N}^{\mathbb{N}} = 2^{\mathbb{N}}$

Now, since $\#\mathbb{N} = \#\mathbb{Q}$

$\#\mathbb{Q}^{\mathbb{N}} = 2^{\mathbb{N}}$

Therefore, $[0,1]^{\mathbb{N}} \sim \mathbb{Q}^{\mathbb{N}}$: equinumerous

$\boxed{\#3}$ Which of the following sets are countable

Def: X is countable if it is finite or $X \sim \mathbb{N}$

(1) $\mathbb{Z}^{\mathbb{N}}$

Obviously not finite

Since $2^{\mathbb{N}}$ is uncountable, neither is $\mathbb{Z}^{\mathbb{N}}$ $(2^{\mathbb{N}} \leq \mathbb{Z}^{\mathbb{N}})$

Also, argument in #2 (3) applies.

(2) $\mathbb{Z}^3 \cup \mathbb{Z}^7$

$\mathbb{Z}^3 = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \sim \mathbb{N}$

same for $\mathbb{Z}^7$. Since both $\mathbb{Z}^3$ and $\mathbb{Z}^7$ are countable, by fact seen in class, so is their union

(3) $\bigsqcup_{n \in \mathbb{N}} \mathbb{Z}^n$

As seen in classe, if $\mathbb{Z}^n$ is countable $\forall n \in \mathbb{N}$ (which is the case, see #3(2)), $\bigsqcup_{n \in \mathbb{N}} \mathbb{Z}^n$ is also countable.

⊛ could be easily proved by induction for $\mathbb{Z}^n$.

(4) $\mathbb{R} \times \mathbb{Q}$

$\mathbb{R}$ is uncountable, so $\mathbb{R} \times \mathbb{Q} \sim 2^{\mathbb{N}} \times \mathbb{N}$ which is not countable.

$\boxed{\#4}$ How many binary relation are there on the set $\{1,2,3\}$?

Since $|\{1,2,3\}| = 3$ and $|\{1,2,3\}| \times |\{1,2,3\}| = 9$,

There are $2^9 = 512$ relations on $\{1,2,3\}$

$\boxed{\#5}$ How many reflexive binary relation?

We want all the sets st $(1,1), (2,2), (3,3)$ are include

Hence, there are $3^2 - 3$ other pair we can choose,

so $2^{3^2-3} = \boxed{64}$ sets w\ the pairs $(1,1)$ $(2,2)$ $(3,3)$ in them.

$\boxed{\#6}$ How many reflexive and symmetric binary relations?

The relation must contain (1,1), (2,2), (3,3) to be reflexive and we also must count the number of relations st $\{(a_i, a_j) \mid 1 \leq i \leq j \leq n\}$

$\Rightarrow 2^{\frac{3^2-3}{2}} = 2^3 =$ 8 reflexive & symmetric relations

$\boxed{\#7}$ How many eq relation?

According to the theorem of Bell number, it suffices to count the number of partitions to obtain the number of eq relations.

$\{\{1\},\{2\},\{3\}\}$
$\{\{1,2\},\{3\}\}$
$\{\{1,3\},\{2\}\}$ $\Rightarrow$ 5 eq relations
$\{\{1\},\{2,3\}\}$
$\{\{1,2,3\}\}$